4.9.2015 UNIT-V - ERROR CONTROL CODING

The physical medium through the messages are transmitted are called channels. The channels are pronto different kind of noises such as lightening, human error, equipment defects, etc. In digital communication system the information transmitted from one end to another depends on,

* Transmitted signal power (E_b)
* Channel Bandwidth

In digital communication systems, there is a limit to value of E_b/N_0, where N_0 is the noise spectral density. Error control coding is needed to achieve good data quality at the receiver with a limit on E_b/N_0 (SNR). They are useful for accurate transfer of information.

Some of ECC (Error control codes) add redundancy in the form of extra symbols to a message prior to transmission.

Advantages of ECC:

* Reduces the required E_b/N_0 for a fixed bit error rate
* Reduces the transmitted power where E_b/N_0 is low.
* Reduces the size of the antenna for radio communication

* Reduces the hardware cost.

Disadvantages of ECC:

* Addition of redundancy increases the transmission Bandwidth.
* Increases the complexity in implentation of decoder.

Channel Coding:

ECC is used to achieve error performance which include considerations of bandwidth & system complexity. Since these ECC's are used to overcome the effects of noise in the channel, the encoding procedure is called channel coding.

Objectives of ECC:

* Capacity to rectify more errors.
* Fast & efficient encoding of message.
* Fast & efficient decoding of message.
* Maximum information rate.

ECC
- Block codes
  - Linear Block codes
  - Cyclic codes
- Convolution codes

Block codes:

The channel encoder accepts information from the source encoder in successive k-bit blocks. For each block, n-k redundant bits are added which are algebrically related in the k^{th} message bit, thereby, producing an overall ~~bit~~ encoded block of 'n'. The n-bit code is called as code word. n - length of the code word. The channel encoder produces the bits at the rate of R_o,

$$R_o = \left(\frac{n}{k}\right) R_s$$

R_s = bit rate of the source.

Code rate, $r = \frac{k}{n}$

Algorithm for Encoding:

Step1: Write the message vector $m_{1\times k}$ (written in rows & columns i.e matrix)

Step2: Compute the parity matrix $P_{k\times n-k}$

Step3: Compute the generator matrix.

$$G = \left[P_{k\times n-k} \; \vdots \; I_{k\times k} \right]$$

$I_{k\times k} \rightarrow$ Identity matrix.

Step4: Compute the code word,

$$C = mG$$

Algorithm for decoding:

Step1: Compute Parity check matrix.

$$H_{n-k \times n} = \left[I_{n-k \times (n-k)} \;\vdots\; P^T_{(n-k)\times n} \right]$$

Step2: Let the received data be 'r'.

Step3: Compute the Syndrome, $S = r H^T$

Step4: Construct the decoding table.

Step5: Find the error pattern 'e'.

Step6: Correct the error by adding the error pattern with the received vector.

length of code word (n)
message bits (k)
(n, k)

1. Construct a (7, 4) Linear Block code having the parity information as: $b_0 = m_0 + m_1 + m_2$; $b_1 = m_0 + m_2 + m_3$; $b_2 = m_1 + m_2 + m_3$. Find the code of word for the message vector, m = 1110 & correct the error that has occured in C_5 position.

Soln:

Bits that we add are called parity

The no: of message bits = k = 4.

The length of the code word, n = 7.

The no: of parity bits = n - k
= 7 - 4
= 3

Encoding:

1) $m_{1\times k} = [1\ 1\ 1\ 0]_{1\times 4}$

2) Parity matrix:

$$P_{k\times n-k} = P_{4\times 3} = \begin{array}{c} \\ m_0 \\ m_1 \\ m_2 \\ m_2 \end{array} \begin{array}{c} \begin{array}{ccc} b_0 & b_1 & b_2 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{4\times 3} \end{array}$$

3) Generator matrix:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 7}$$

7) $C = mG$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}_{1\times 4} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{4\times 7}$$

$$= [1\oplus 1\oplus 1\oplus 0 \quad 1\oplus 0\oplus 1\oplus 0 \quad 0\oplus 1\oplus 1\oplus 0 \quad 1\oplus 0\oplus 0\oplus 0$$
$$0\oplus 1\oplus 0\oplus 0 \quad 0\oplus 0\oplus 1\oplus 0 \quad 0\oplus 0\oplus 0\oplus 0]$$

$$C = \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array}$$
$$C = [\underbrace{1 \quad 0 \quad 0}_{\text{Parity bits}} \quad \underbrace{1 \quad 1 \quad 1 \quad 0}_{\text{Message bits}}]$$

Due to the channel noise, the error has occurred in c_5th position. ∴ The received vector,

$$r = [1\ 0\ 0\ 1\ 1\ \underline{\underline{0}}\ 0]$$

[error: 1 is changed to 0]

Decoding: 1) $r = [1\ 0\ 0\ 1\ 1\ 0\ 0]$

$$H = \left[I_{3\times 3} \;\vdots\; P^T_{3\times 4} \right]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$S = rH^T$

$$S = [1\ 0\ 0\ 1\ 1\ 0\ 0]_{1\times 7} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{7\times 3}$$

$$s = [1 \oplus 0 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \quad 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \quad 0 \oplus 0 \oplus 0 \oplus 0 \oplus 1 \oplus 0 \oplus 0]$$

$$s = [1 \quad 1 \quad 1]_{1\times 3}$$

Decoding table:

Syndrome	Error pattern
0 0 0	0 0 0 0 0 0 0
1 0 0	1 0 0 0 0 0 0
0 1 0	0 1 0 0 0 0 0
0 0 1	0 0 1 0 0 0 0
1 1 0	0 0 0 1 0 0 0
1 0 1	0 0 0 0 1 0 0
$\boxed{1\ 1\ 1}$	$\boxed{0\ 0\ 0\ 0\ 0\ 1\ 0}$
0 1 1	0 0 0 0 0 0 1

s ←

Syndrome = H^T

$\Rightarrow e = 0000010$

5) The error pattern,

$e = 0000010$

6) $c = r + e$

$r = 100\ 1100$

$e = 0000010$

$$c = \begin{array}{r} 100\ 1100 \\ (+)\ 000\ 0010 \\ \hline 100\ 1110 \end{array} \rightarrow c$$

∴ We have received the original data transmitted

$c = 1001110$

Hamming distance:

It is defined as no. of locations in which the elements of two code vectors differ. It is represented by d(m, q)

$m = [1 0 1 0 0 1 0]$

$q = [\underset{1}{0}\ \underbrace{0 1}_{\text{same bit}}\ \underset{1}{1}\ \underbrace{0 1}_{\text{same bit}} 0]$ $\therefore d(m, q) = 2$

Hamming weight:

It is defined as no. of non-zero elements in a code vector. It is denoted as w_m.

Minimum hamming distance, "d_{min}" is the smallest hamming weight of all the non-zero code vectors.

The total no. of 1's.

$m = [1 0 1 0 0 1 0]$

$\Rightarrow$ Hamming weight $w_m = 3$.

$q = [0 0 1 1 0 1 0]$

$w_m = 3$

Find the minimum hamming weight among the following.

$m = [1 1 1\ 0 0 1 1] = 5$

$q = [0 0 1 1\ 0 1 1] = 4$

$v = [1 1 0 0\ 0 1 0] = 3$

$c = [0 0 0 0\ 0 0 0] = 0$

For finding the minimum haming weight the non-zero code vector, which contain non-zero element should only we consider,

$d_{min} = 3$

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Syndrome & its properties.

A syndrome 'S' is defined as a $1 \times n-k$ for a (n,k) linear block code. It contains the information above the error pattern and is used for error detection & correction. It is calculated by the formula,

$$S = r H^T$$

where, r - received vector.

H^T - parity check matrix

If $S \neq 0$, then $r \neq c$ and says that error has occured.

$S = 0$, $r = c$, and says that no error has occurred.

Property 1: The syndrome depends only on the error pattern and not on the transmitted code word.

we know that,

$$c = mG$$

$$r = c + e$$

$$S = r H^T$$

$$S = (c+e) H^T$$

$$S = cH^T + eH^T$$

$$S = mG H^T + eH^T$$

As per linear block code technique (LBC), $GH^T = 0$

$$\therefore S = eH^T$$

Property 2:

All the error patterns that differ by a code word have the same syndrome.

c_i = new error pattern

Proof: Let $e_i = e + c_i$

we know that, $S = e_i H^T$ from property - I.

$$S = (e + c_i)\, H^T$$
$$= eH^T + c_i H^T \qquad \because c_i = m_i G$$
$$= eH^T + m_i G H^T$$

By LBC, $GH^T = 0$

$$\therefore S = eH^T$$

1) All the linear block codes satisfy the condition, $G^T H = H^T G = 0$

2) Closure Property:

States that sum of any two code words is a code word.

Consider a (6,3) LBC, with a parity check matrix $H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

Find the generator matrix and all possible code words

Solu:

The length of code word, $n = 6$

The no: of message bits, $K = 3$

The no: of parity bits, $n - k = 6 - 3$
$= 3$

As the no: of message bits is '3', the possible message vectors are,

$m = 2^3 = 8$

$m =$ 000
001
010
011
100
101
110
111

$$H_{3\times7} = \left[\, I_{3\times3} \;\vdots\; P^T_{3\times4} \,\right]$$

$$H = \begin{array}{c} \quad\; P \qquad\quad\; I \\ \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$P^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

To find Generator matrix:

$$G = \left[\, P_{k\times n-k} \;\vdots\; I_{k\times k} \,\right]$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Note:

From the parity check matrix, H, it is clear that, P^T & P are the same ∴ The parity check matrix G, will be same

If message contains all zeros then the code words are also all zeros.

To find code words:

$$C = m\,G$$

$$c_1 = [0\ 0\ 1] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 1\ 1\ 0\ 0\ 1]$$

$$c_2 = [0\ 1\ 0] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [0\ 1\ 1\ 0\ 1\ 0]$$

$$c_3 = [0\ 1\ 1] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 0\ 0\ 0\ 1\ 1]$$

$$c_4 = [1\ 0\ 0] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= [1\ 0\ 1\ 1\ 0\ 0\ 1]$$

0 0 1
0 1 1
0 1 1
0 0 0
0 1 0
0 0 1

1 0 0
0 0 0
1 0 0
1 0 0
0 0 0
0 0 0
0 0 0

$$C_5 = [1\,0\,1]\begin{bmatrix}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$= [0\;1\;0\;1\;0\;1]$$

$$C_6 = [1\,1\,0]\begin{bmatrix}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$= [1\;1\;0\;1\;1\;0]$$

$$C_7 = [1\,1\,1]\begin{bmatrix}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$$

$$= [0\;0\;1\;1\;1\;1]$$

To find syndrome:

Consider, C_7 & C_5 bit in it,

$C_7 = [0\,0\,1\,1\,1\,1]$

C_5th bit of C_7

$\Rightarrow r = [0\,0\,1\,1\,1\,\underline{\underline{0}}]$

$$S = rH^T$$

$$= [0\,0\,1\,1\,1\,0]\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$S = [0\;\;0\;\;1]_{1\times 3}$$

Decoding table:

Syndrome	Error Pattern
0 0 0	0 0 0 0 0 0
1 0 1	1 0 0 0 0 0
0 1 1	0 1 0 0 0 0
1 1 1	0 0 1 0 0 0
1 0 0	0 0 0 1 0 0
0 1 0	0 0 0 0 1 0
0 0 1	0 0 0 0 0 1

e = 000001

c = r + e

c = 0 0 1 1 1 0 → r

0 0 0 0 0 1 → e

0 0 1 1 1 1 = c_7

∴ c_7 = 001111

∴ we received the original signal transmitted.

Cyclic Codes:

It is a code in which cyclic shift of a code word produces another code word.

Advantages of cyclic codes:

Easy to encode

They possess a well-defined mathematical structure, thereby providing a efficient decoding scheme.

Properties:

Linear Property:

The sum of any two code words is also a code word.

Cyclic property:

The cyclic shift of a code word is also a code word (ie)

Consider a (n, k) block code having a code word $\{c_0, c_1, \ldots c_{n-1}\}$

It is said to be cyclic iff $\{c_{n-1}, c_0, c_1 \ldots c_{n-2}\}, \{c_{n-2}, c_{n-1}, \ldots c_{n-3}\}$ is also a code word.

Representation of a code word as a polynomial:

$$c_x = c_0 + c_1 x + \ldots\ldots + c_{n-1} x^{n-1}$$

Advantages of representing a code word as polynomial:

1. These are algebric codes, hence the algebric operations such as addition, subtraction, multiplication & division are very simple.

2. The position of the bits are represented with the help of powers of x in a polynomial (ie) $x^{n-1} \to MSB$; $x^0 \to LSB$.

Message Bit as polynomial:

$$m(x) = m_0 + m_1 x + m_2 x^2 + \dots + m_{k-1} x^{k-1}$$

Parity bit as a polynomial:

$$b(x) = b_0 + b_1 x + \dots + b_{n-k-1}$$

$$\therefore C(x) = b(x) + x^{n-k} m(x)$$

Generator Polynomial: [G(x)]

It is a polynomial of degree $(n-k)$, which is a factor of $x^n + 1$

$$G(x) = 1 + \sum_{i=1}^{n-k-1} g_i x^i + x^{n-k}$$

$$g_i = 0 \text{ or } 1$$

Generator matrix: $[G_{k \times n}]$

The generator matrix is constructed using 'k' polynomials (ie), $g(x), x g(x), \dots x^{k-1} g(x)$

$$G_{k \times n} = \begin{bmatrix} g(x) \\ x g(x) \\ \vdots \\ x^{k-1} g(x) \end{bmatrix}$$

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Parity check polynomial:

$$h(x) = 1 + \sum_{i=1}^{k-1} h_i x^i + x^k$$

$h_i = 0$ or 1

Parity check matrix:

$H_{n-k \times n}$

This matrix is constructed using $(n-k)$ polynomials where, the polynomials are,

$$x^k h(x^{-1}),\ x^{k+1} h(x^{-1}), \ldots\ x^{n-1} h(x^{-1})$$

These are used as rows in matrix represented as shown,

$$\begin{bmatrix} x^k h(x^{-1}) \\ \vdots \\ x^{n-1} h(x^{-1}) \end{bmatrix}$$

Encoding Algorithm for Cyclic Codes:

Step1: Factorize x^n+1 into irreducible polynomials of degree 1.

Step2: Find the primitive polynomial of degree m. It satisfies the condition, $n = 2^m - 1$

Step3: Assign, one of the primitive polynomial, as generator polynomial, $g(x)$.

Step4: Assign the other remaining polynomials as parity check polynomial as, $h(x)$.

Step5: Write the message polynomial, $m(x)$

Step6: Find the product of $m(x) \cdot x^{n-k}$

Step7: Divide $m(x) \cdot x^{(n-k)}$ by $g(x)$.

$m(x)\, x^{(n-k)} / g(x)$

Step 8: Assign the remainder as $b(x)$

Step 4: Find the code polynomial,

$$C(x) = b(x) + x^{n-k} \cdot m(x)$$

1. Find the code word for the message 1001 by constructing a (7,4) cyclic code. (Do this for generator poly)

Sol:

$n = 7$

$k = 4$

1) $x^n + 1$

$\Rightarrow x^7 + 1$

$= \underbrace{(1+x)}_{m=1} \underbrace{(1+x+x^3)}_{m=3} \underbrace{(1+x^2+x^3)}_{m=3 \to \text{degree}}$

2) $n = 2^m - 1$

for, $m = 1$;

$n = 2^1 - 1 = 2-1 = 1 \neq 7 \neq n$

$\therefore (1+x)$ is not primitive,

for $m=3$, $n = 2^3 - 1 = 8-1 = 7 = n$

$\therefore 1+x+x^3$ is primitive polynomial.

Similarly,

$1+x^2+x^3$, is also a primitive polynomial.

as, $2^m - 1 = 2^3 - 1 = 7 = n$

3) $g(x) = 1+x+x^3$ (Select the one with lower order)

4) $h(x) = (1+x)(1+x^2+x^3)$

$= 1+x^2+x^3+x+x^3+x^4$

$= 1+x+x^2+2x^3+x^4$

Neglect $2x^3$, since the coeffecients of x should be either '1' or 0.

$\therefore h(x) = 1+x+x^2+x^4$.

5) $m = 1001$

$\therefore m = 1\times x^0 + 0\times x^1 + 0\times x^2 + 1\times x^3$

$= 1+x^3$

6) $m(x) \cdot x^{n-k}$

$= (1+x^3) \cdot x^{(7-4)} = (1+x^3)(x^3)$

$= x^3 + x^6$

7) $m(x)(x^{n-k}) / g(x)$

$$= \frac{x^3+x^6}{1+x+x^3}$$

$$\begin{array}{r|l}
 & x^3 - x \\
\hline
1+x+x^3 & x^3 + x^6 \\
 & x^3 + x^6 + x^4 \\
 & (-) \quad (-) \quad (-) \\
\hline
 & -x^4 \\
 & -x^4 - x^2 - x \\
 & (+) \quad (+) \quad (+) \\
\hline
 & x^2 + x
\end{array}$$

8) $b(x) = x^2+x$

9) $c(x) = b(x) + m(x)(x^{n-k})$

$= x+x^2+x^3+x^6$

$\therefore c(x) = (0\,1\,1\,1\,0\,0\,1)$

Find the generator matrix and parity check matrix for the above problem.

Solu:

Generator matrix:

$$\begin{bmatrix} g(x) \\ x(g(x)) \\ x^2 g(x) \\ x^3 g(x) \\ \vdots \\ x^{k-1}(g(x)) \end{bmatrix} = \begin{bmatrix} 1+x+x^3 \\ x+x^2+x^4 \\ x^2+x^3+x^5 \\ x^3+x^4+x^6 \end{bmatrix} \begin{bmatrix} g(x) \\ x\,g(x) \\ x^2 g(x) \\ x^3 g(x) \end{bmatrix}$$

$$\begin{array}{c} \qquad\quad P \qquad\qquad\qquad I \\ G = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \end{array}$$

Since we did not get the identity matrix

∴ The generator matrix is not systematic.

Decoding of Cyclic codes:

The code word $c_0, c_1, \ldots c_{n-1}$ is transmitted over a noisy channel which results in a received word $r_0, r_1, \ldots r_{n-1}$

This received word can be represented as a polynomial

$$r(x) = r_0 + r_1 x + \ldots + r_{n-1} x^{n-1}$$

$$\frac{r(x)}{g(x)} = q(x) + \frac{s(x)}{g(x)}$$

$$r(x) = q(x)g(x) + s(x)$$

where, $s(x) \rightarrow$ remainder of Syndrome.

1. Find the parity check matrix for the above problem.

Solu:

$$H(x) = \begin{bmatrix} x^{k} h(x^{-1}) \\ x^{k+1} h(x^{-1}) \\ \vdots \\ x^{n-1} h(x^{-1}) \end{bmatrix}$$

[n = 7
k = 3]
[n - k = 4]

we know that,

$$h(x) = 1 + x + x^2 + x^4 \quad \text{(from previous problem)}$$

$$h(x^{-1}) = \left(1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^4}\right)$$

$$= \frac{x^4 + x^3 + x^2 + 1}{x^4}$$

$$H(x) = \begin{bmatrix} x^4 \left(\frac{x^4 + x^3 + x^2 + 1}{x^4}\right) \\ x^5 \left(\frac{x^4 + x^3 + x^2 + 1}{x^4}\right) \\ x^6 \left(\frac{x^4 + x^3 + x^2 + 1}{x^4}\right) \end{bmatrix}$$

$$= \begin{bmatrix} x^4 + x^3 + x^2 + 1 \\ x^5 + x^4 + x^3 + x \\ x^6 + x^5 + x^4 + x^2 \end{bmatrix}$$

$$\therefore H(x) = \left[\begin{array}{ccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{array}\right] \quad \begin{array}{cc} I & P \end{array}$$

$$\Rightarrow H(x) = [\underbrace{I_{3\times3}}_{I_{n-k}} \mid P]$$

$\therefore H(x)$ is not systematic

Structure of decoder:

No. of flip flops = n-k

no. of modulo-2 = n-k-1

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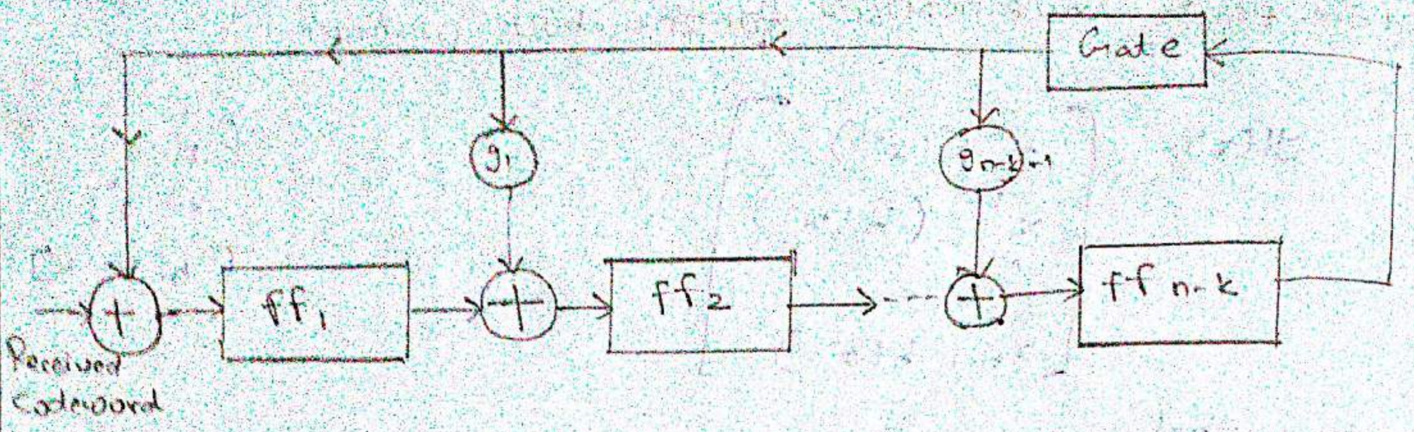


1. Find the syndrome using syndrome calculator & decode the cyclic code (7, 4)

Modulo-2 adders - Adders performing Ex-OR operation

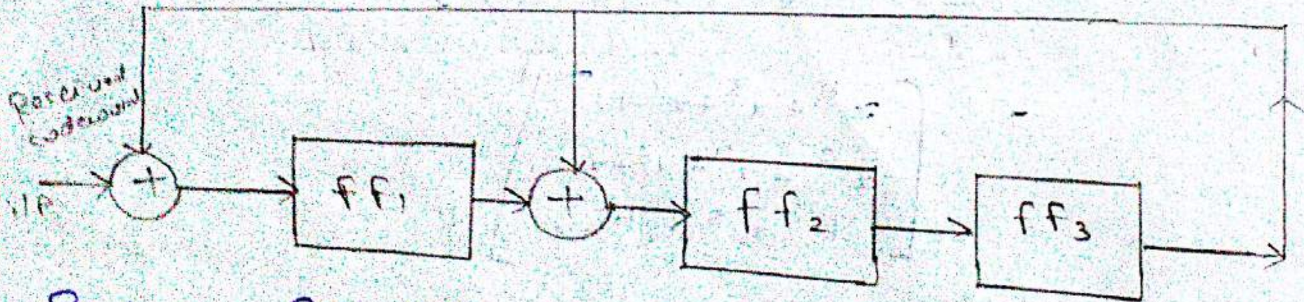
Solu:

for (7, 4)

no: of flip flops = 7 - 4 = 3

no: of adders = 7 - 4 - 1 = 2

1) adder
2) flipflop
3) last flip flop o/p is given to the 1st adder.



Syndrome Calculator:

I/P	I/P to ff1 O/P of ff3 + I/P	I/P to ff2 O/P of ff3 + O/P of ff1	I/P to ff3 O/P of ff2
0	0	0	0
1	1 ⊕ 0 = 1	0 ⊕ 0 = 0	0
0	0 ⊕ 0 = 0	1 ⊕ 0 = 1	0
0	0 ⊕ 0 = 0	0 ⊕ 0 = 0	1
0	0 ⊕ 1 = 1	1 ⊕ 0 = 1	0
1	0 ⊕ 1 = 1	0 ⊕ 1 = 1	1
1	1 ⊕ 1 = 0	1 ⊕ 1 = 0	1
0	1 ⊕ 0 = 1	1 ⊕ 0 = 1	0

1st assume from LSB of the codeword

o/p → previous o/p
→ add with previous o/p

Code word: 0111001 (LSB)

If C_3 is error,

r = 0110001

$S = [1\ 1\ 0]$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Symbol	Error pattern
000	0000000
101	1000000
010	0100000
101	0010000
110	0001000
111	0000100
011	0000010
001	0000001

$$r = 0110001$$
$$(+)\ e = 0001000$$
$$c = 0111001$$

Properties of syndrome in cyclic codes:

1) The syndrome of the received word polynomial is also the syndrome of the corresponding error polynomial.

2) The syndrome of the received word polynomial x, is the syndrome for cyclic shifted received word polynomial.

3) The syndrome polynomial is identical to the error polynomial assuming that the errors are confined to n-k parity bits.

Convolution codes:

By default assume G_1 the 1st one if no generator sequence is given

Given the generator sequence $G_1 = 1 + x^2$, $G_2 = 1 + x + x^2$. And the message bits at 10101. Find the code word with the help of encoder diagram.

$G_1 = 1x^0 + 0x^1 + 1x^2 = \{1\,0\,1\}$

$G_2 = 1x^0 + 1x^1 + 1x^2 = \{1\,1\,1\}$

$k = 3$

Solu:

Convert the generator sequence G_1, G_2 into bit format. (write the co-efficients of the polynomial given above)

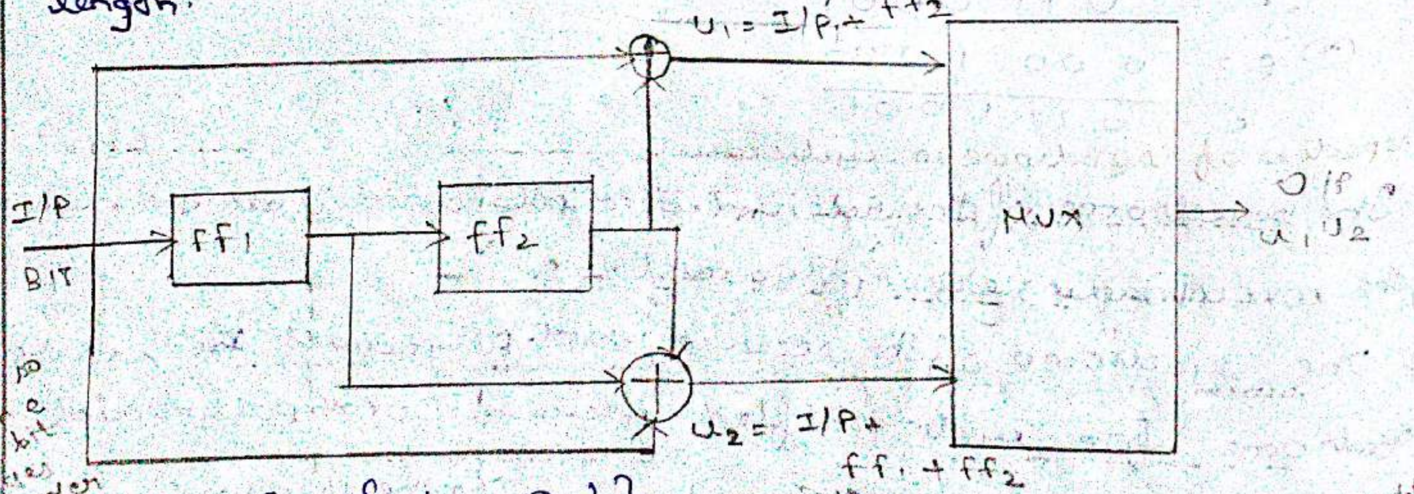
$G_1 = \{1\,0\,1\}$

$G_2 = \{1\,1\,1\}$

No: of adders: $k-1$

Constraint Length $= 3 = k$

To draw the encoder diagram flip flop, modulo-2 adder & multiplexer is required. The no: of flip flop & the no: of modulo-2 adders is equal to $k-1$, where k is the constraint length.



$G_1 = \{\underset{I/P}{1}\ \underset{ff_1}{0}\ \underset{ff_2}{1}\}$

Previous Value

State Table:

Message bits (I/P)	State ff1	ff2	u_1 I/P + FF2	u_2 I/P + FF1 + FF2	Mux O/P u_1	u_2
Initial	0	0				
1	1	0	$1 \oplus 0 = 1$	$1 \oplus 0 \oplus 0 = 1$	1	1
0	0	1	$0 \oplus 0 = 0$	$0 \oplus 1 \oplus 0 = 1$	0	1
1	1	0	$1 \oplus 1 = 0$	$1 \oplus 0 \oplus 1 = 0$	0	0
0	0	1	$0 \oplus 0 = 0$	$0 \oplus 1 \oplus 0 = 1$	0	1
	1	0	$1 \oplus 1 = 0$	$1 \oplus 0 \oplus 1 = 0$	0	0

General representation

$G_1 = \{\underset{I/P}{1}\ \underset{ff_1}{0}\ \underset{ff_2}{1}\}$

$\Rightarrow u_1 = I/P + ff_2$

$G_2 = \{\underset{I/P}{1}\ \underset{ff_1}{1}\ \underset{ff_2}{1}\}$

$\Rightarrow u_2 = I/P + ff_1 + ff_2$

∴ Codeword = {11, 01, 00, 01, 00}

In order to find the code word using a "code tree or Trellis" the state table that defines all the possibilities of the i/p & the states should be calculated.

2. Find the codeword for message 10101 having $G_1 = 1 + x^2$ & $G_2 = 1 + x + x^2$ using codetree & trellis.

Soln:

Since the convolutional code for a single bit is a dibit there are four possibilities for a single bit. (00, 01, 10, 11). Each of these states are assigned a variable as,

a → 00
b → 01
c → 10
d → 11

State Table:

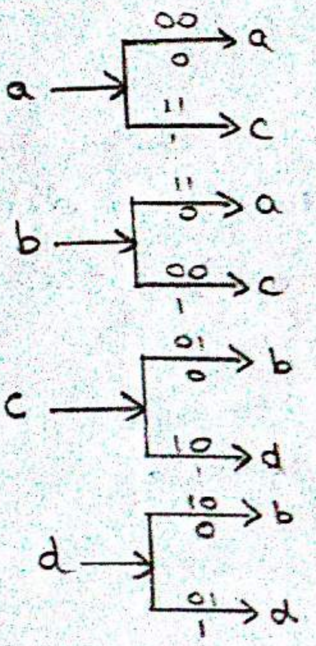
Message bit	Present state			Next state		Output	
	ff_1	ff_2		ff_1	ff_2	u_1 (i/p ⊕ ff2)	u_2 (i/p ⊕ ff1 ⊕ ff2)
0	0	0	a	0	0 → a	0	0
Ex-OR 1	0	0	a	1	0 – c	1	1
Ex-OR 0	0	1	b	0	0 – a	1	1
1	0	1	b	1	0 – c	0	0
0	1	0	c	0	1 – b	0	1
1	1	0	c	1	1 – d	1	0
0	1	1	d	0	1 – b	1	0
1	1	1	d	1	1 – d	0	1

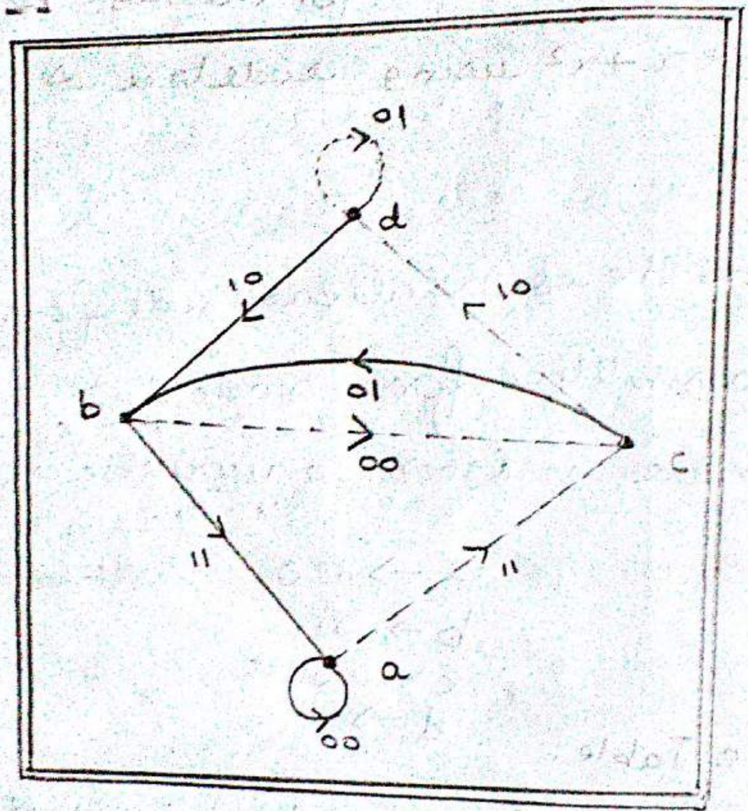
State Diagram:

Consider 'a' tobe as your initial state. If your initial bit is zero then draw a bold line tonext state if the initial bit is one draw a dotted line to next state

Message bit is 0 ______

Message bit is 1 - - - -





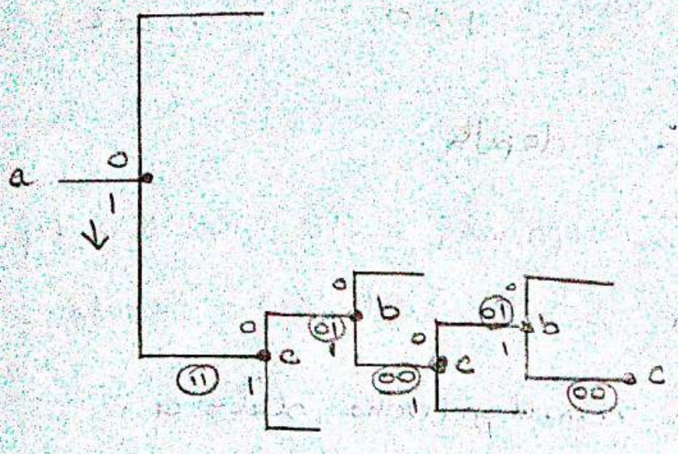
Code tree:

Start with a initial state 00 & the two possibilities of message 0's & 1's and representing using bifocation.

i.e) when message (MSB) is '0' the upper part of the tree is only drawn, similarly when the message (MSB) is '1' the lower part of the tree is only drawn. And the o/p for each transition of state is written on the line connecting the 2 states.

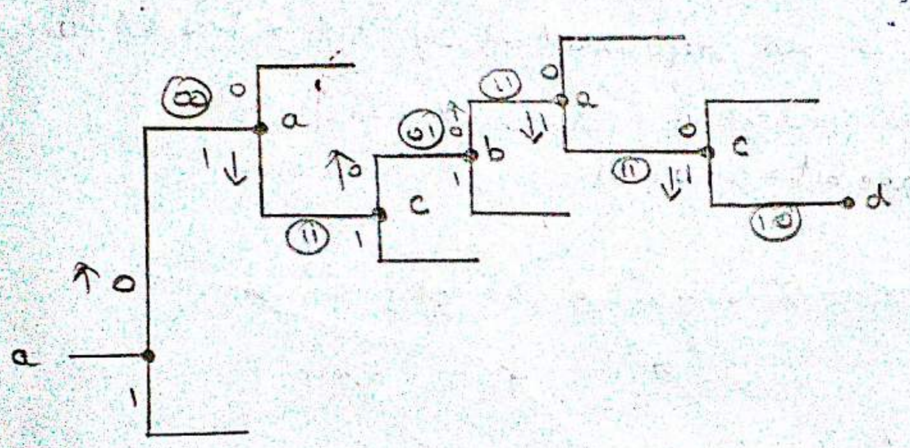
For 16-Mark complete the Code tree.

Message = 10101



∴ Codeword = {11, 01, 00, 01, 00}

3. Draw a code tree for message 010011



∴ codeword = {00, 11, 01, 11, 11, 10}

Trellis:

It is another method to find the code word for a convolutional code. A trellis is a tree like structure with remerging branches. A code branch produced by i/p message '0' is drawn as a solid line. A code branch produced by i/p message '1' is drawn as dashed line.

A trellis is a more instructive structure than a code tree. It brings out explicitly that the convolutional encoder is a finite state machine. The trellis contains 'L+K' levels where 'L' is the length of the incoming message and 'K' – constraint length of code.

The levels of trellis are, j = 0, 1, ... , L+K-1

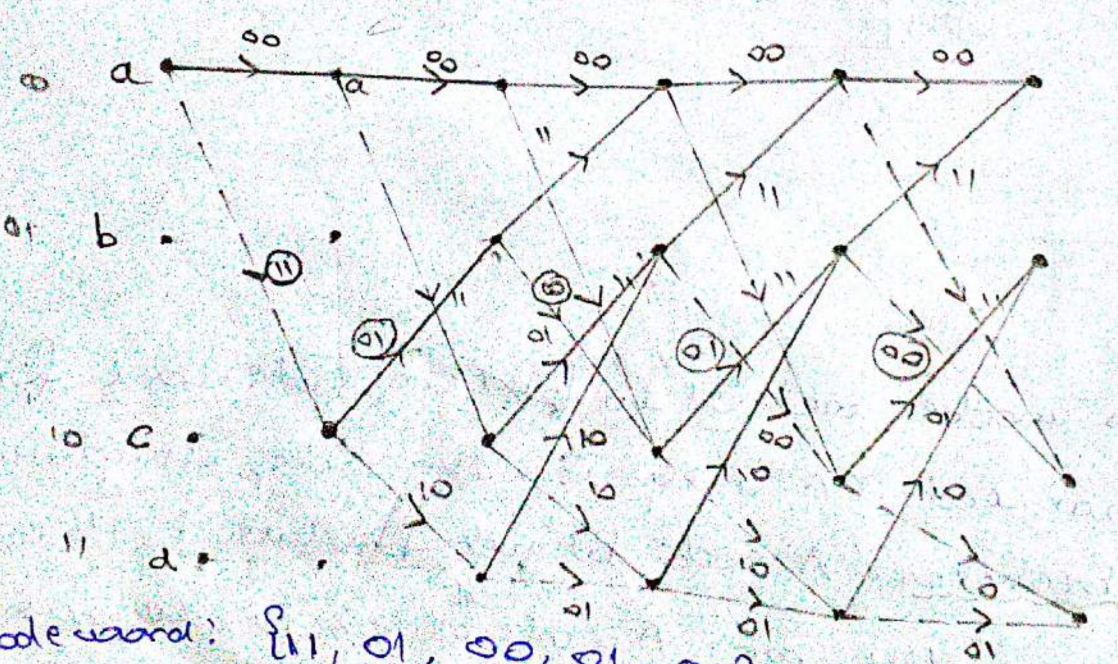
The level 'j' is referred as depth.

The first 'K-1' levels corresponds to the encoders departure from the initial stage 'a' & the last 'K-1' levels, corresponds to the encoders written to the state 'a'.

All the states of the encoder are labelled as a, b, c, d.

The left nodes represents the current state & right node represents next state. A transition from one state to another for i/p '0' is represented by "solid branch" & for a i/p '1' it is represented as "dashed line"

Message bit = 10101



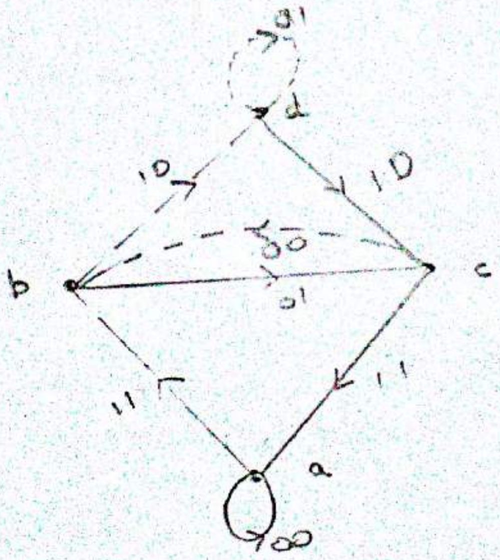
Codeword: {11, 01, 00, 01, 00} → Message = 10101

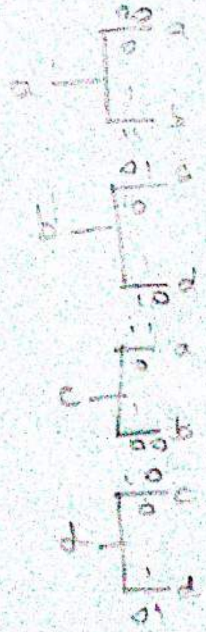
Codeword: {11, 10, 10, 00, 01} → Message = 11010

Construct a state table, state diagram, code tree, trellis for a → 00, b → 10, c → 01, d → 11. Find codeword for message 1011.

Message bits	Present state			Next state		Output	
	ff_1	ff_2		ff_1	ff_2	u_1 i/p + ff_2	u_2 i/p + ff_1 + ff_2
0	0	0	a	0	0 - a	0	0
1	0	0	a	1	0 - b	1	1
0	1	0	b	0	1 - c	0	1
1	1	0	b	1	1 - d	1	0
0	0	1	c	0	0 - a	1	1
1	0	1	c	1	0 - b	0	0
0	1	1	d	0	1 - c	1	0
1	1	1	d	1	1 - d	0	1

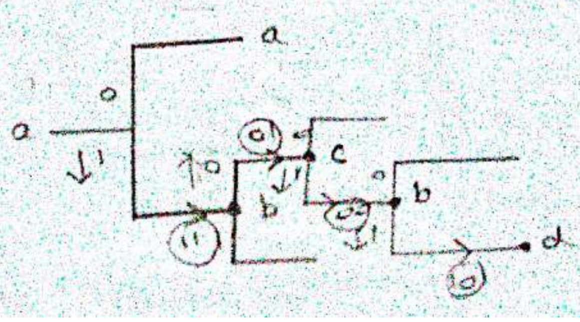
State Diagram:



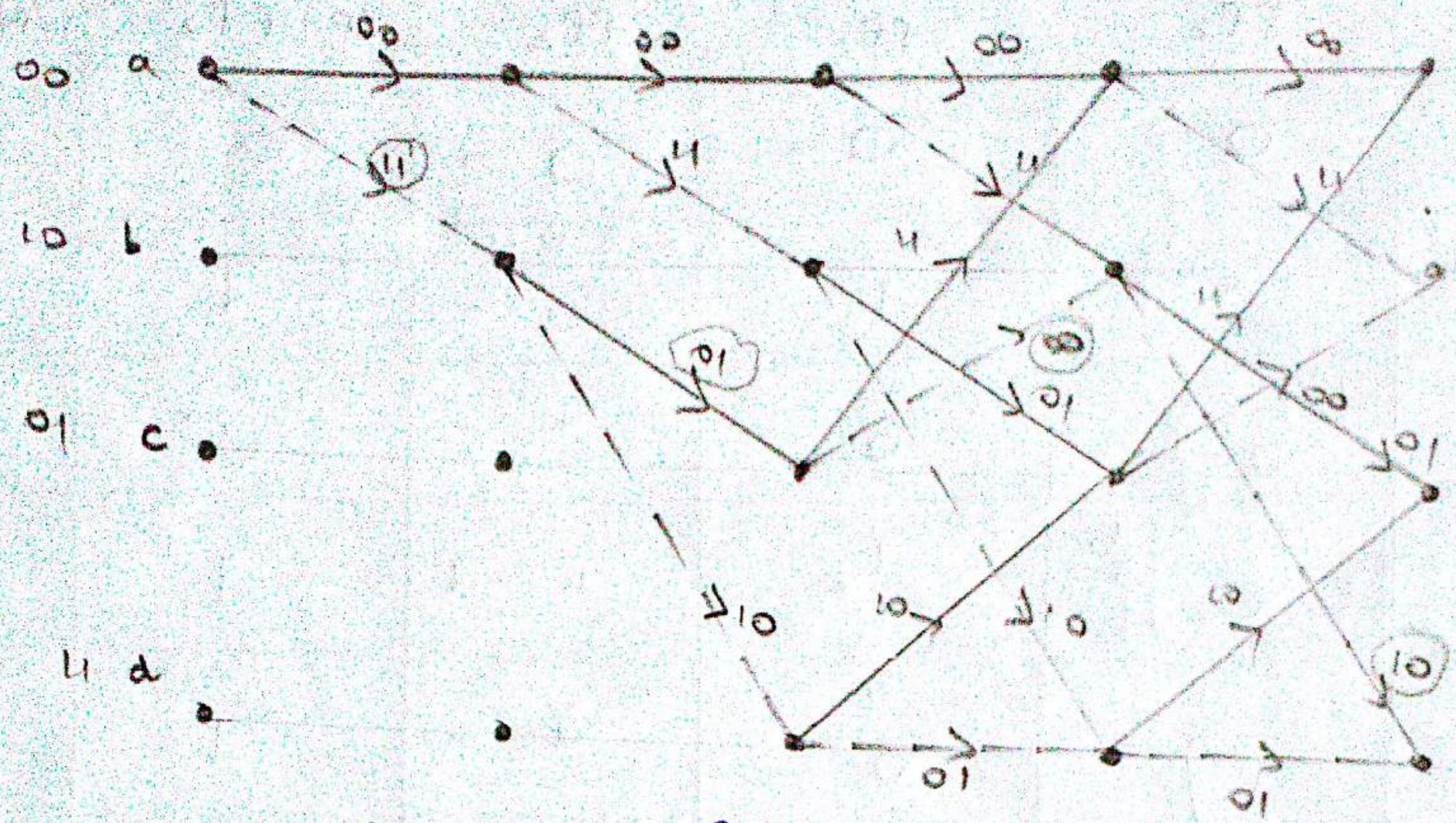


Code tree:- Message . 1011

∴ Codeword = {11, 01, 00, 10}



Trellis:



∴ Codeword = {11, 01, 00, 10}